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RAN-2303080302020002

M.Sc. (Sem. - 2) Examination September - 2025

Mathematics (Complex Analysis - 202)

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) There are five questions in this question paper
(3) Answer all questions
(4) Figure to the right indicates full marks of the questions.

Seat No.:

Student's Signature

Q. 1 Attempt any Two **14**

A. Show that the function $f(z) = 2z + z^2$, $|z| < 1$ is univalent in the domain and show that $\lim_{z\to\infty} \frac{1}{z^2} = 0$

B. For the function $f(z)$ defined by $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ then show that the C-R equations are satisfied at (0,0) but the function is not differentiable at (0,0).

C. Derive the polar form of C-R equations.

Q. 2 Attempt any Two **14**

A. Transform the Laplace equation $u_{xx} + u_{yy} = 0$ into polar coordinates, and show that $u(r, \theta) = e^{-\theta} \cos(\text{In } r)$, $r > 0, 0 < \theta < 2\pi$ is harmonic. Also, find the harmonic conjugate of $u(r, \theta)$ and the corresponding analytic function.

B. Let $f(z)$ be a piecewise continuous function defined on a contour γ then

$\left| \int_\gamma f(z)dz \right| \leq ML$, where L is the length of the contour γ and $M = sup_{z\in\gamma} |f(z)|$.

Also, Find the upper bound of the integral $\left| \int_c \frac{(z^2+3)e^{iz} Logz}{z^2-2} dz \right|$; C = {Z:Z = $2e^{i\theta}$, $0 \leq \theta \leq \frac{\pi}{3}$ }

C. State and prove Cauchy - Goursat theorem and evaluate $\int_c \frac{e^{-z}}{(z-2\pi)^4} dz$, where C: $4x^2 + 9y^2 = 36$.

Q. 3 Attempt any Two **14**

A. Define the radius of convergence of power series and show that

$1 + \left(\frac{a.b}{1.c}\right)z + \left(\frac{a(a+1)b(b+1)}{1.2.c.(c+1)}\right)z^2 + ...$ has unit radius of convergence.

B. Let the radius of convergence of power series $\sum_{n=0}^{\infty} a_n z^n$ be R, $R > 0$ then show that the function defined by the equation $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is analytic in $|z| < R$ and moreover $f'(z) = \sum_{n=1}^{\infty} n a_n z^{n-1}$

C. Prove that:

(i) $\frac{1}{z^2} = \sum_{n=0}^{\infty} (n+1)(z+1)^n$

(ii) $\frac{1}{(1-z^2)^2} = \sum_{n=0}^{\infty} (n+1)z^{2n}$

Q. 4 Attempt any Two **14**

A. Let $f(z)$ be analytic in $|z| < 1$ and $|f(z)| \leq 1$ on $|z| < 1$ then $\left|\frac{f(z)-f(z_0)}{1-\overline{f(z_0)}\, f(z)}\right| \leq \left|\frac{z-z_0}{1-\bar{z}_0 z}\right|$ for any z, z_0 inside the unit disc.

B. State and prove Laurent theorem

C. Evaluate two Laurent series expansion in the power of z of the function $f(z) = \frac{1}{z(1+z^2)}$

Q. 5 Attempt any Two **14**

A. The function $f(z)$ has a pole of order m at z_0 iff $f(z)$ can be written in the form $f(z) = \frac{\varphi(z)}{(z-z_0)^m}$, where $\varphi(z_0) \neq 0$, $\varphi(z)$ is analytic at z_0.

B. If f is analytic within and on a positively oriented simple closed contour γ and does not vanished on γ then prove that $\frac{1}{2\pi i}\int_{\gamma} \frac{f'(z)}{f(z)} dz = N - P$; where N is the number of zeros and P is the number of poles of f which lies inside γ, both zeros and poles being counted according to their multiplicity and evaluate

$\int_{c} \frac{f'(z)}{f(z)} dz$ for $f(z) = z^5 - 3iz^2 + i - 1$; where all zeros of $f(z)$ lies in C.

C. Show that the only singularities of an analytic function $f(z)$ are poles of order 1 and 2 at $z = -1$ and $z = 2$ with residues 1 and 2, respectively at these poles. Determine $f(z)$ if it also satisfies the conditions $f(0) = \frac{7}{4}$ and $f(1) = \frac{5}{2}$
